

BLUE PRINT FOR QUESTION PAPER PATTERN
COURSE-IV, REAL ANALYSIS

Unit	TOPIC	S.A.Q(including choice)	E.Q(including choice)	Total Marks
I	Real Number System and Real Sequence	2	2	30
II	Infinite Series	1	2	25
III	Limits and Continuity	1	2	25
IV	Differentiation and Mean Value Theorem	2	2	30
V	Riemann Integration	2	2	30
	TOTAL	8	10	140

S.A.Q. = Short answer questions (5 marks)

E.Q. = Essay questions (10 marks)

Short answer questions : 5 X 5 M = 25 M

Essay questions : 5 X 10 M = 50 M

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Total Marks = 75 M
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CBCS/ SEMESTER SYSTEM
(w.e.f. 2020-21 Admitted Batch)
B.A./B.Sc. MATHEMATICS
COURSE-IV, REAL ANALYSIS

Time: 3Hrs

Max.Marks:75M

SECTION - A

Answer any **FIVE** questions. Each question carries **FIVE** marks **5 X 5 M=25 M**

1. Prove that every convergent sequence is bounded.
2. Show that $\lim_{n \rightarrow \infty} \left(\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \dots + \frac{1}{(n+n)^2} \right) = 0$.
3. Test the convergence of the series $\sum_{n=1}^{\infty} (\sqrt[3]{n^3 + 1} - n)$.
4. Examine for continuity of the function f defined by $f(x) = |x| + |x - 1|$ at $x=0$ and 1 .
5. Show that $f(x) = x \sin \frac{1}{x}$, $x \neq 0$; $f(x) = 0$, $x = 0$ is continuous but not derivable at $x=0$.
6. Verify Rolle's theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ on $[1, 3]$.
7. If $f(x) = x^2 \forall x \in [0, 1]$ and $p = \left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}$ then find $L(p, f)$ and $U(p, f)$.
8. Prove that if $f: [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$ then f is R-integrable on $[a, b]$.

SECTION -B

Answer **ALL** the questions. Each question carries **TEN** marks. **5 X 10 M = 50 M**

9.(a) If $S_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!}$ then show that $\{S_n\}$ converges.

(OR)

(b) State and prove Cauchy's general principle of convergence.

10.(a) State and Prove Cauchy's nth root test.

(OR)

(b) Test the convergence of $\sum \frac{x^n}{x^n + a^n}$ ($x > 0, a > 0$).

11.(a) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be such that

$$f(x) = \frac{\sin(a+1)x + \sin x}{x} \text{ for } x < 0$$
$$= c \text{ for } x = 0$$

$$= \frac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}} \text{ for } x > 0$$

Determine the values of a, b, c for which the function f is continuous at $x=0$.

(OR)

(b) Define uniform continuity, If a function f is continuous on $[a, b]$ then f is uniformly continuous on $[a, b]$

12.(a) Using Lagrange's theorem, show that $x > \log(1+x) > \frac{x}{(1+x)} \forall x > 0$.

(OR)

(b) State and prove Cauchy's mean value theorem.

13.(a) State and prove Riemann's necessary and sufficient condition for R- integrability.

(OR)

(b) Prove that $\frac{\pi^3}{24} \leq \int_0^\pi \frac{x^2}{5+3\cos x} dx \leq \frac{\pi^3}{6}$.